Equal Temperament via the "Marpurg" Quasi Equal Temperament

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Here is yet another idea which technicians who normally tune using an Electronic Tuning Device (ETD) will find useful, easy to understand and remember. This temperament sequence will be a practical method for anyone attempting the PTG Tuning Exam or when aurally pitch raising or tuning a piano which is substantially out of tune. One of its features is that the sequence remains within the familiar F3-F4 octave where the inharmonicity remains the most constant and beat rates the most predictable.

While it is meant to be a rough tuning sequence, it is possible by following these very simple steps to achieve a temperament octave on the very first pass which will score a perfect 100 on the tuning exam. At the very least, it moves each note into a position where any known techniques for refining the temperament may then be used.

The name Marpurg is invoked because after the preliminary steps, the result is actually one of the historical temperaments known as the Marpurg-Neidhardt Quasi Equal Temperament often called simply, "Marpurg". (See my previous PTG Journal article on that subject for full details. Six of the notes of the temperament are then easily moved to produce a true Equal Temperament (ET). This is all done without counting beats! There is no guessing; only one estimate of the F3-A3 M3 and that estimate is either confirmed or self corrected by easily heard comparisons of beat rates, not by actually counting beats. After the F3-A3 M3 estimate, each step progresses towards a very nearly perfect ET.

The process begins with the construction of the four contiguous Major thirds (henceforth may be abbreviated as CM3; Major thirds are abbreviated as M3) as you may have seen in many contemporary temperament sequences. While easy to understand for some, it is daunting for many. I have heard many technicians say, "I wouldn't know how to tune all of those thirds!" Therefore, I will hopefully make tuning the contiguous M3s far easier for you to understand and accomplish with a high degree of perfection than ever before.

You will not need to use any of the fine tuning checks you may know during this sequence. During either a pitch raise or tuning a piano which has been detuned for the PTG Tuning Exam, all notes would be too far off pitch to use those techniques. It is best to use this simple sequence which brings every pitch exactly to or at least very close to where it should be and then to use fine tuning techniques to improve the sequence if time permits.

**Tips on Tuning the Contiguous M3s from F3 to A4**

You have probably heard of the 4:5 Ratio of Contiguous M3s. The interval, Major third is abbreviated as, M3. Remember that any interval is either beatless, wider than beatless, or narrower than beatless. The M3 is a widened interval. In theory, the M3 is approximately 14 cents wide of beatless. Widening a M3 will make it beat faster; narrowing a M3 will make it beat slower. The terms beatless, pure, just and just intonation may be used interchangeably.

The 4:5 Ratio of Contiguous M3s is a scientific principle that was rarely known before Dr. Al Sanderson developed the first practical ETD for tuning pianos. Not only is it an extremely valuable tool for identifying errors within the temperament as well as anywhere in the piano where M3s can easily be heard, it serves as the ultimate tool for precisely and unequivocally dividing the temperament octave into three equal parts. Once the octave is divided in that way, there will be four anchor notes from which the rest of the temperament octave can hardly go wrong, no matter which sequence you may choose to tune the rest of the notes. The 4 notes which divide the octave then become anchor notes upon which the rest of the temperament is based. Contiguous M3s are defined as any two M3s which share a common note between them. Example: F3-A3 and A3-C#4. This is distinguished from chromatic M3s which are those next to each other. Example F3-A3 and F#3-A#3.
Of course, the 4:5 ratio does not mean four beats per second versus five beats per second; instead, it means that for every four beats of the lower M3, there are five beats in the upper M3 of any two contiguous M3s, anywhere on the piano where they can be heard. Naturally, this applies only to ET regardless of the amount of stretch in the temperament octave. It does not apply to any other kind of temperament.

Do not be concerned if you are not sure what the 4:5 ratio of contiguous M3s should sound like. You need only know that it would be deemed a small difference. It is not the very smallest discernible difference which can be heard but neither is it an obviously large discrepancy. Indeed, it is far easier to hear 4:5 the ratio of contiguous M3s than it is to hear the very small difference between contiguous minor thirds (m3) which have a ratio of 5:6. The ratio of difference between any two chromatic M3s or m3s is extremely small, approximately 15:16.

So, in comparison the small difference found in the 4:5 ratio of contiguous M3s is one that is quite easily heard, identifiable, and most importantly, controllable. If any two intervals beat exactly the same, they are known as equal beating. If any two contiguous M3s beat equally or very nearly so, the relationship is not correct and certainly, if the lower or bottom M3 of any two contiguous M3s beats faster than the upper or top M3, the relationship is very incorrect because it is inverted from what it should be.

**Important clues for tuning and correcting the four initial Contiguous M3s**

Normally, the F4-A4 M3 beats too fast to be useful when comparing chromatic M3s. In that range of the piano, M6s and M10s are much easier to hear. However, when constructing these four initial contiguous M3s, the top F4-A4 M3 is an essential tool in determining whether the bottom three contiguous M3s, F3-A3, A3-C#4 and C#4-F4 are correct. The bottom three may sound acceptable by themselves but the top F4-A4 M3 can serve to identify the very smallest error. Don’t make the mistake of working only with the set of three CM3s, F3-A3, A3-C#4 and C#4-F4. The top set, F4-A4 must be included to ensure that the bottom three are actually correct. The important difference is that there are two octaves, F3-F4 and A3-A4, not just one, the F3-F4. These two octaves and the set of four CM3s will provide accuracy for these five pitches, F3, A3, C#4, F4 and A4 that can be considered as reliable as is humanly possible by aural tuning.

The top F4-A4 M3 sounds very fast, at perhaps the limit of perception or discernibility. Therefore, if it beats gently and is easily heard, it is too slow and as you work through this process, it will become clear that if the top F4-A4 M3 is too slow, it will mean that the bottom F3-A3 M3 is also too slow because both F3 and F4 are too sharp by the same amount. When that is the case, there will be difficulty placing C#4. It cannot be placed so that A3-C#4 and C#4-F4 have the proper 4:5 ratio. The likelihood is that C#4-F4 and F4-A4 will beat nearly the same or even be inverted when A3-C#4 and F3-A3 have the proper 4:5 ratio between them. The F3-F4 octave must be considered as a unit. If F3 is moved, so must F4 be moved by exactly the same amount.

If the top F4-A4 M3 is beating too fast, its beats may be so fast as to not be heard; it will be an indiscernible blur and the interval would likely have a “sour” sound. In that case, there would be an obviously large difference in beating between the contiguous M3s C#4-F4 and F4-A4 when F3-A3 and A3-C#4 have the proper 4:5 ratio between them. The A3-C#4 and C#4-F4 M3s will then either beat too nearly the same or be inverted.

Very important clues to how large or small the error is between any two contiguous M3s are equal (or very similar) beating and inverted beating. If any two contiguous M3s beat exactly the same or very nearly the same, there is a small error, perhaps about one cent. If the bottom of any two contiguous M3s beats faster than the upper, the error is larger, perhaps 2 cents or more. The more inverted any two contiguous M3s are in their beating, the larger the error.
If you are a technician who normally uses an ETD and are struggling with aural tuning concepts in your quest to pass the PTG tuning exam or simply trying to learn aural tuning for its benefits and you spot a small error, visualize what making a 1 cent or so correction would be like using your ETD. The pattern would be moving only slowly and slightly. It would take only the slightest application of tuning hammer technique and test blows to make that correction. Once you've applied the techniques, listen again to the relationship of the two contiguous M3s to determine if you accomplished that very small correction or made a correction that was too large.

As an exercise in listening, use your ETD to tune the three notes F3, A3 and C#4. Listen to that small difference between the rates of beating in the two contiguous M3s. Now, set your ETD on A3 and tune A3 just one cent flatter. You'll surely hear the beat rates and ratio change! Try also sharpening A3 from its normal position and try sharpening and flattening F3 and C#3 similarly so that you become accustomed to hearing the significant difference that only a one cent change makes in the beating of a M3 and the ratio of beating of two contiguous M3s.

In the following sequence, Major thirds are abbreviated as M3, fourths are abbreviated as P4 and fifths as P5. The "P" stands for "perfect" even though fourths and fifths in ET are always tempered. The word, perfect, comes from music theory, not tuning theory. You will note that there are no other intervals used such as sixths (M6) or minor thirds (m3).

The sequence is easy to use and remember because after the initial series of contiguous M3s, only P4s and P5s are tuned and you don't have to guess at their tempering! Each P4 or P5 is initially tuned beatless (also known as "pure" or "just") and then is made to beat exactly the same as another P4 or P5 to which it is compared. It is easy for anyone to cause any two intervals to beat exactly the same and does not involve counting or timing the beats against a clock. While the sequence may seem ponderous to read in its written form, once you have studied and understood it, you will see how easy it actually is. There is an abbreviated summary which follows the detailed sequence.

The Sequence for Tuning the Initial Contiguous M3s

1. Tune A4 to an aural pitch source such as a tuning fork.
2. Tune A3 to A4, a beatless sounding octave, approaching from the wide side of beatless, just to insure the octave is not any amount narrow.
3. Estimate the F3-A3 M3. Any reasonable estimate will work. If you have no idea what the usual 7 beats per second specification sounds like, simply sharpen and flatten F3 until you hear a moderate amount of beating (the F3-A3 M3 must be a widened interval, not narrowed), neither obviously slow or obviously too fast and "sour" sounding. Anywhere within that range is a close enough estimate at this point.
4. Tune F4 from F3, a beatless sounding octave, again approaching from the wide side of beatless to avoid a narrow octave. Both F3 and F4 may have to be slightly readjusted afterwards but the next note you tune will tell you whether F3 and F4 need adjustment or not and whether they need to be flattened or sharpened.
5. Play the A3-C#4 M3. Sharpen or flatten C#4 until the A3-C#4 M3 is a widened interval and beats at first exactly the same as the F3-A3 M3. At this point, you know that the C#4 must be sharpened slightly because contiguous M3s can never beat exactly the same as one another. Sharpen C#4 so that the A3-C#4 M3 beats a small amount faster than the F3-A3 M3 does presently.
6. Now, listen to the sequence of contiguous M3s: F3-A3, A#-C#4, C#4-F4 and F4-A4. Do all M3s progress slightly in their rate of beating? If not, the clue to which way to now flatten or sharpen both F3 and F4 will be in any error found in this slight progression of beating from one M3 to the next.
7. If F3 and F4 are too sharp, the C#4-F4 M3 will beat much faster than the A3-C#4 M3. The F4-A4 M3 will also be the same or slower than the C#4-F4 M3, an inverted ratio. If you have the two contiguous M3s C#4-F4 and F4-A4 beating the same or very nearly so, only a very slight adjustment flatter of both F3 and F4 is needed. If the rate of beating is inverted, the C#4-F4 M3 beats faster than the F4-A4 M3, a larger adjustment flatter of both F3 and F4 is needed. If the top M3, F4-A4 beats too slowly, then it means that the bottom M3, F3-A3 also beats too slowly.

8. If F3 and F4 are both too flat, the A3-C#4 and C#4-F4 will beat nearly the same if the error is slight and will be inverted if the error is larger. The C#4-F4 M3 will reveal a little too little difference from A3-C#4 if the error is slight. The C#4-F4 M3 will beat slower than the A3-C#4 M3, an inverted ratio if the F3 and F4 are too flat. If the top M3, F4-A4 beats too rapidly, then it means that the bottom M3, F3-A3 also beats too rapidly.

9. After adjusting the F3 flatter or sharper as is indicated, immediately retune F4 to F3, again approaching a beatless sounding octave from the wide side. Then, listen to the F3-A3 M3 and adjust C#4 if need be so that the A3-C#4 M3 beats slightly faster than the F3-A3 M3. Generally, if both F3 and F4 are flattened or sharpened, C#4 must also be moved, if however so slightly, in the same direction as both F3 and F4.

Listen to the sequence of contiguous M3s again and make any small adjustments necessary to achieve the small and smooth progression of beat rates. When you believe you have it, it will be within the range of tolerance of the PTG exam for each note! A one cent change in any of these notes from when they are correct will be very obviously wrong. So, when the sequence sounds correct and acceptable to you as that small progression from each M3 to the next, you are well within an acceptable range, if not at absolute perfection.

10. The F3, A3, C#4, F4 and A4 having been tuned, return to the bottom three notes of that sequence, F3, A3 and C#4 and ask yourself which notes within the F3-F4 octave could you tune either a P4 or a P5 from each of these notes? The answer is from F3: A#3 and C4. From A3: D4 and E4. From C#4: F#3 and G#3.

11. Temporarily tune A#3 a beatless P4 and C5 a beatless P5 from F3.

12. Temporarily tune D4 a beatless P4 and E4 a beatless P5 from A3.

13. Temporarily tune F#3 beatless P5 and G#4 a beatless P4 from C#4.

At this point, there are three more notes which have not yet been touched, G3, B3 and D#4.

14. Temporarily tune G3 from C4 a beatless P4. Now play the G3-D4 P5 and notice that it beats strongly as a narrowed P5. Flatten G3 until both the G3-C4 P4 and G3-D4 P5 beat exactly the same. Both intervals will still beat more noticeably at this point than they will upon completion of the ET sequence.

15. Temporarily tune B3 from F#3 a beatless P4. Now, listen to the B3-E4 P4 and notice that it beats strongly as a widened P4. Sharpen B3 until the F#3-B3 P4 and the B3-E4 P4 beat exactly the same. Both intervals will still beat more noticeably at this point than they will upon completion of the ET sequence.

16. Temporarily tune D#4 from A#3 a beatless P4. Now, listen to the G#4-D#4 P5 and notice that it beats strongly as a narrowed P5. Sharpen D#4 until both the A#3-D#4 P4 and G#3-D#4 P5 beat exactly the same. Both intervals will still beat more noticeably at this point than they will upon completion of the ET sequence.
As of this moment, you have moved all 13 notes of the F3-F4 octave and have arrived at the Marpurg temperament. If you play all of the consecutive M3s you will notice that they sound apparently the same as ET. However, if you play the M6s, you will notice a problem or two and if you play all of the P4s and P5s, half of them will be beatless and the other half will beat too strongly; in fact, twice as much as in a true ET.

ET has no beatless P4s or P5s. Since you initially tuned the CM3s F3-A3, A3-C#4 and C#4-F4 the same as for any other ET sequence, they are of course, correct but now, G3, B3 and D#4 are also correct. That provides a whole tone scale, F3, G3, A3, B3, C#4, D#4 and F4 where all of the tuning pins are in a straight line and all of these pitches can be considered very reliable. What remains is a parallel whole tone scale where all of the tuning pins are also in a straight line but each pitch is theoretically 2 cents sharp or flat from true ET.

17. Go back to the notes from which you first tuned beatless P4s and P5s. Those are F3, A3 and C#4. Those notes are very reliable. Remember that now G3, B3 and D#4 are also very reliable. As you go to each note that you initially tuned as a P4 or P5, think about which note from the G3, B3 or D#4 list to which you could compare the initial beatless P4 or P5.

18. You temporarily tuned A#3 from F3 as a beatless P4. Which note from the G3, B3 and D#4 list could you compare A#3 as either a P4 or P5? The answer is D#4. Listen to the beatless F3-A#3 P4 and then to the A#3-D#4 P4 which will beat strongly as a widened P4. Sharpen A#3 until both the F3-A#3 and A#3-D#4 P4s beat exactly the same. Both will now have the barely perceptible beats of true ET.

19. Listen to the beatless F3-C4 P5 and then to the G3-C4 P4 which will beat strongly as a P4. Flatten C4 until both the F3-C4 P5 and the G3-C4 P4 beat exactly the same. Both will now have the barely perceptible beats of true ET.

20. Go back to A3. You temporarily tuned D4 and E4 from it both as a beatless P4 and P5. Listen to the beatless A3-D4 and then to the G3-D4 P5 which will beat strongly as a narrowed P5. Sharpen D4 until both the A3-D4 P4 and the G3-D4 P5 both beat exactly the same. Both will now have the barely perceptible beats of true ET.

21. Listen now to the beatless A3-E4 P5 and then to the B3-E4 P4 which will beat strongly as a widened P4. Flatten E4 until both the A3-E4 P5 and the B3-E4 P4 beat exactly the same. Both will now have the barely perceptible beats of true ET.

22. Go back to C#4. You temporarily tuned F#3 and G#3 from it as a beatless P4 and P5. Play the beatless F#3-C#4 P5 and then to the F#3-B3 P4 which will beat strongly as a widened P4. Sharpen F#3 until both the F#3-B3 P4 and the F#3-C#4 P5 beat exactly the same. Both will now have the barely perceptible beats of true ET.

23. Listen to the beatless G#3-C#4 P4 and then to the G#3-D#4 P5 which will beat strongly as a narrowed P5. Flatten G#3 until both the G#3-C#4 P4 and the G#3-D#4 P5 beat exactly the same. Both will now have the barely perceptible beats of true ET.

The sequence is now complete. You should hear a good, if not perfect sounding ET. If errors are heard, you may use any techniques you may know of to correct them. You may also use another sequence you know and begin the process again using all of the checks you may know. Before using any checks or starting another sequence, re-examine the initial set of CM3s, F3-A3, A3-C#4, C#4-F4 and F4-A4. Even though these were considered very reliable, any pitch may have drifted slightly or the A3-A4 and F3-F4 octaves may not be exactly the same size. This is the time to use octave checks.
If you are at a tuning exam and time is of the essence, rough in the remaining octaves in the midrange at this point, using the same technique as you did initially with the F3-F4 and A3-A4 octaves. Each note to tune will be alternately sharp or flat on a PTG tuning exam detuned piano. To rough in the octave, find the point where the octave is obviously too wide and beating strongly, then narrow the octave just until you hear the beat stop. That will provide a good estimate for the octave at this point. You can fine adjust the octaves if you have time after re-examining the temperament octave.

For the purposes of this study, a 4:2 octave is the goal. The test note for a 4:2 octave is the M3 below the bottom note of the octave. To test the A3-A4 octave, play first the F3-A3 M3 and then the F3-A4 M10. Listen carefully to the beats of both intervals. When they both beat exactly the same, a 4:2 octave is proven. If the F3-A3 M3 beats slightly faster than the F3-A4 M10, the octave is slightly narrow. Flatten A3 slightly and test again for absolute equal beating of both intervals. If the F3-A4 M10 beats slightly faster than the F3-A3 M3, the octave is slightly wide. Sharpen A3 slightly and test again for absolute equal beating of both intervals.

For the initial set of CM3s to be absolutely correct and reliable, both pairs of octaves, F3-F4 and A3-A4 must be proven to be exactly 4:2 octaves. Therefore, after determining that the A3-A4 octave is correct, check F3-F4 the same way. If you have adjusted A3, it may or may not have upset the delicate balance of the four CM3s. Check the F3-F4 octave as it now stands. The test note is C#3.

To test the F3-F4 octave, play first the C#3-F3 M3 and then the C#3-F4 M10. Listen carefully to the beats of both intervals. When they both beat exactly the same, a 4:2 octave is proven. If the C#3-F3 M3 beats slightly faster than the C#3-F4 M10, the octave is slightly narrow. Flatten F3 slightly and test again for absolute equal beating of both intervals. If the C#3-F4 M10 beats slightly faster than the C#3-F3 M3, the octave is slightly wide. Sharpen F3 slightly and test again for absolute equal beating of both intervals.

Once you have proved that both the F3-F4 octaves are exactly the same size or width, perfect 4:2 octaves, you must check C#4 to see if the F3-A3 M3 and A3-C#4 have the proper 4:5 ratio of beating. If not, adjust C#4 until the ratio is correct. If that spoils the arrangement of the CM3s, it means that both F3 and F4 are either slightly sharp or flat. If that is the case, you must adjust F3 sharper or flatter as needed and then re-tune F4 to F3 and then again check to make sure the octave is correct using the octave test. Then check and adjust C#4 if necessary and review the entire arrangement of CM3s again. All of these adjustments described in the last four paragraphs will likely be very small. Even if there is no time for them, the entire midrange is likely to be within passing range of the PTG tuning exam.

After determining that the initial CM3s are correct upon re-examination, you can go through any sequence that has been previously familiar to you. Most likely, as you arrive at each note of the sequence, you will find it to already be correct or only slightly in error. If an error is discovered, correct it at that point and use any interval checks that you may know to prove the correction. Bear in mind that the F3, A3, C#4, F4 and A4 will always be the most reliable pitches from which you can check. If only a few very slight errors are discovered, you can make spot corrections with the knowledge that the initial set of CM3s serve as anchors; evenly spaced pitches from which the entire rest of the temperament can be checked and verified.

After checking and verifying the temperament octave, check the rest of the notes in the midrange by first verifying a reasonable sounding octave, then checking corresponding P4s and P5s, then checking the rapidly beating intervals in any ways known to you.
The ET Via Marpurg Summary Sequence

After studying the ET via Marpurg sequence in Part 1 and 2 of this article, use the following summary as a quick reference until you have memorized the sequence.

1. Tune A4 to an A-440 aural pitch source.
2. Tune A3 from A4, a beatless octave.
3. Estimate the F3-A3 M3 to approximately 7 beats per second.
4. Tune F4 from F3, a beatless octave.
5. Temporarily tune the A3-C#4 M3 to beat exactly the same as the F3-A3 M3, then sharpen C#4 so that the A3-C#4 M3 beats slightly faster than the F3-A3 M3, a 4:5 ratio of beating between the F3-A3 M3 and the A3-C#4 M3.
6. Test the four contiguous M3s (CM3s) from F3 to A4 for a 4:5 ratio of beating between and among all four CM3s. If a correction is needed, move F3 first then re-tune F4 to F3. Now test the 4:5 ratio of beating of F3-A3 and A3-C#4. Adjust C#4 if necessary. Retest the CM3s from F3 to A4. Repeat these procedures if necessary until all CM3s from F3 to A4 bear a 4:5 ratio of beating.
7. Temporarily tune A#3 from F3, a beatless P4.
8. Temporarily tune C4 from F3, a beatless P5.
10. Temporarily tune E4 from A3, a beatless P5.
11. Temporarily tune F#3 from C#4, a beatless P5.
12. Temporarily tune G#3 from C#4, a beatless P4.
13. Temporarily tune G3 from C4, a beatless P4, then compare the G3-C4 P4 to the G3-D4 P5 and flatten G3 until both the G3-D4 P5 and G3-C4 P4 beat exactly the same.
14. Temporarily tune B3 from F#3, a beatless P4, then compare the F#3-B3 P4 to the B3-E4 P4 and sharpen B3 until both the F#3-B3 P4 and the B3-E4 P4 beat exactly the same.
15. Temporarily tune D#4 from A#3, a beatless P4, then compare the A#3-D#4 P4 to the G#3-D#4 P5 and sharpen D#4 until both the G#3-D#4 P5 and A#3-D#4 P4 beat exactly the same.
16. Compare the F3-A#3 P4 to the A#3-D#4 P4. Sharpen A#3 until both the F3-A#3 P4 and A#3-D#4 P4 beat exactly the same.
17. Compare the F3-C4 P5 to the G3-C4 P4. Flatten C4 until both the F3-C4 P5 and the G3-C4 P4 beat exactly the same.
18. Compare the A3-D4 P4 to the G3-D4 P5. Sharpen D4 until both the G3-D4 P5 and the A3-D4 P4 beat exactly the same.
19. Compare the A3-E4 P5 to the B3-E4 P4. Flatten E4 until both the A3-E4 P5 and the B3-E4 P4 beat exactly the same.
20. Compare the F#3-C#4 P5 to the F#3-B3 P4. Sharpen F#3 until both the F#3-B3 P4 and the F#3-C#4 P5 beat exactly the same.
21. Compare the G#3-C#4 P4 to the G#3-D#4 P5. Flatten G#3 until both the G#3-D#4 P5 and the G#3-C#4 P4 beat exactly the same.

Disclaimer

The ET via Marpurg sequence is intended as a rough tuning idea to move each note very quickly and easily to a point where a fine tuning using typically known checks can be used more effectively. The fact is that it still produces a quasi equal temperament (quasi means almost). The reason is because in fact, P4s and P5s do not beat equally in true ET. The P4 actually beats about one third faster than the P5 in theoretical terms and when the temperament octave is stretched beyond the point where it sounds beatless, there is yet more of a difference between P4s and P5s.
You may have heard that some technicians use a wider than 4:2 octave. Indeed, most ETD programs do work with a slightly wider octave. However, for the purposes of this study, the 4:2 octave is used to keep the study as simple as possible for those new to aural tuning. Using a 4:2 octave will work just as effectively on the PTG tuning exam as any slightly wider choice of octave. Therefore, it is suggested to start with this concept. Once you have learned aural tuning sufficiently to experiment with a wider choice of octave, then will be the time to do it, after you have already become an RPT!

If these directions are followed literally, there are some notes which will be exactly the same as in ET: the F3, A3, C#4, F4 and A4 because they are tuned exactly the way any other sequence for tuning which uses those contiguous M3s has them tuned. The rest of the notes still fall within the one cent tolerance of the PTG Exam, some are only off by nearly immeasurable amounts; others are off by only about half the tolerance for an error. Only the D#4 comes remotely close to being an error at -0.78 cents flat. That is still 0.22 cents from being counted as an error and possibly even being aurally perceived as an error.

Although one could bear this in mind when using the sequence and tune any P4s proportionately faster than P5s, it is not really recommended to do that. It is better to use this equal beating method and move each note to its approximate position and then solve any audible discrepancies afterwards. When performing a pitch raise or tuning a very out of tune piano, refining the temperament after each step would be counterproductive.

If you are a person who normally uses an ETD and are attempting the PTG tuning exam, this sequence alone, if done reasonably well and the remaining notes in Octave 3 and 4 are tuned as beatless octaves to the temperament range, the results by only following these steps, can achieve a perfect score. During Part 1 of that exam, if you have time remaining and can correct any flaws, by all means do so and remember to tune any corresponding octaves as beatless to those changes.

If you know and are comfortable using any of the myriad of more complex checks that are available and have the time to use them and are sure you won’t make things worse instead of better, do it! It is always possible that some notes you tuned may have drifted. It is also possible that an interval you intended to tune beatless was actually slightly tempered, perhaps even in the wrong direction. That is what the fine tuning checks are for but again, when every note is far from being where it will eventually be, these checks aren’t particularly useful. Any slight errors you may have made can skew the results and that is why you should refine the temperament after roughing it in if time permits.

Below are the theoretical deviations from true ET of the ET via Marpurg as calculated by Professor Owen Jorgensen RPT:

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<tr>
<td>D#</td>
<td>-0.78</td>
</tr>
<tr>
<td>E</td>
<td>-0.50</td>
</tr>
<tr>
<td>F</td>
<td>0.00</td>
</tr>
<tr>
<td>F#</td>
<td>+0.60</td>
</tr>
<tr>
<td>G</td>
<td>+0.56</td>
</tr>
<tr>
<td>G#</td>
<td>-0.05</td>
</tr>
<tr>
<td>A</td>
<td>0.00</td>
</tr>
<tr>
<td>A#</td>
<td>+0.16</td>
</tr>
<tr>
<td>B</td>
<td>+0.56</td>
</tr>
</tbody>
</table>
Here is a graph by Jason Kanter which shows all properties of the ET via Marpurg. The display appears more irregular by sight than the temperament would actually sound by ear. Note that the largest deviation is -0.78, below the 1.0 threshold of error on the PTG tuning exam.
Should I use a wider initial octave than 4:2?

Some technicians studying for the tuning exam have expressed concern about octave width or stretch. The fact is that not all exam master tunings have the same amount of stretch in them. However, it will not matter whether your tuning has more or less stretch than the master tuning; it will only be important that your tuning is consistent with itself. Therefore, if you feel uncomfortable tuning with octaves that do have a slight beat in them, don’t risk it! It is a better idea to tune what seem to be beatless 4:2 octaves and be safe. If you have time to use checks which prove the octaves are correct, so much the better. Once Part 1 of the exam is completed, you have the right to use your ETD in part 2 to tune the outer octaves.

How to Test Yourself

As practice for aural tuning of a piano which has been detuned the way it will be for a tuning exam, I suggest the following: Program your ETD normally, then offset it 4.0 cents flat and beginning on C3, tune every other note (in whole steps) until you reach A#4. Then return to C#3 and offset the device 4.0 cents sharp. Starting with C#3, tune every other note until you reach B4. The Midrange will now be detuned in virtually the same way as you will encounter at a tuning exam.

You can also test and score yourself for the temperament and entire midrange much the way it would be done on an actual tuning exam using your ETD’s exam program. First, program your ETD as you normally would. If you know how to prescribe it to tune 4:2 octaves in the midrange, you can use that feature but if not, it is not essential. Tune the midrange from C3 to B4 as carefully as possible.

Then, following the directions in your ETD manual, open the exam pages. Carefully read each pitch from C3 to B4 which you have tuned. Play each note softly and if your piano has a sostenuto pedal, use it to keep the damper open while you carefully stop the pattern for each pitch. Enter each pitch as read on the “Master” tuning page. This page may be called by various names. Consult your ETD manufacturer if you are unsure of the name of the page. When all pitches are entered, you will have a reference tuning from which you may judge your aural tuning at any time.

To judge your aural tuning results, enter each pitch which you have tuned aurally from C3 to B4 on the “test” or “examinee” page. Again, the title of this page may vary from one manufacturer to another. Consult the manual or manufacturer if you are unsure. Use F3 as the bottom note of the temperament octave for scoring purposes according to your manual. You can test only the temperament octave if you wish or the entire midrange. However, you cannot judge only the initial CM3s until you have at least completed the entire temperament octave.

Consultants: Owen Jorgensen RPT and Jim Coleman RPT
Editing consultant: Jerry Viviano
Sources: Handbook for Tuning the Equal Beating Temperaments and Tuning by Owen Jorgensen RPT

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