Achieving 21st Century Standards of Excellence in Tuning

Aural Tuning Tests

for

2:1, 4:2 and 6:3 Type Octaves

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There have recently been many questions posed about octave sizes, the differences between them and how to tune them. The fact is, that you can construct an Equal Temperament within any size octave, a 2:1, 4:2 or 6:3 or even a wider than a 6:3 octave or a slightly narrow octave if that is your goal. It is simply the consensus of opinion today that a compromise between a 4:2 and 6:3 octave is the optimum. The very slight beat you will hear in such an octave will make your P5 a little purer and your P4 and M3 a little faster than what is theoretically correct and what all the old books on tuning say is correct.

None of these books that I know of, say anything other than to make a *pure* sounding octave. In reality, an octave can only be truly pure, meaning having no audible beat, between any two matching sets of *coincident partials* (explanation to follow) but there is indeed a small range within which an octave may sound beatless or pure to the ear. From what we know today however, tuning an octave from A4 to A3 that sounds pure to the ear, when starting the tuning process with no rapidly beating interval checks, would most probably create an octave anywhere from a 2:1 to a 4:2 type. This article will explain in detail in how to tune any size octave you choose and how to know the difference.

Before going any further, let's define just what these numbers, said to be octave *types,* are. They are ratios, yes, but they are not the equivalent of each other as they would seem to be theoretically. This has been a source of confusion to some because ratios are used in defining other intervals. A 5th is a 3:2 and a M3 is a 5:4 for example. These are figures which also point to *partials* which *coincide* and indeed a 5th can often have more than one set of audible *coincident partials* (the definition to follow) but a M3 and a 5th are not usually discussed as *types* the way octaves are, only whether they are tempered or not and to what degree.

Theoretically, an octave has a ratio of 2:1 and any multiple thereof, such as 4:2, 6:3 and beyond are the equivalent, merely figures unreduced to the lowest common denominator. The ratio of 2:1 indicates simply that the upper note of the octave theoretically has twice the frequency of vibrations per second as the lower. But when discussing octave *types,* these figures indicate what are known as *coincident partials*. The words, *partial, harmonic* and *overtone* all refer to the same phenomenon and are generally interchangeable but piano technicians usually prefer to use the word *partial* (used as a noun) when discussing tuning.

A partial is one of the higher and usually fainter (but not always) sounds that we hear along with the *fundamental* which is the pitch we think about and recognize when any given string of the piano is struck. The fundamental tone or pitch is also considered to be the *first partial.* Theoretically, an upper or higher partial is an exact multiple of the fundamental pitch, such as twice the frequency, two and a half times the frequency, three times the frequency, etc. Most people, even musicians do not perceive (hear) partials. Even many piano technicians go their entire career saying that they do not hear them. Others consider hearing partials as essential to tuning. Not to worry however, because when we hear *beats* in any interval, we are hearing partials which are mismatched or not exactly in tune with each other. Indeed, the very essence of aural tuning depends upon the ability of the tuner to *perceive and control beats*.

Any string of the piano has a series of partials which, if transcribed to look like music, appear to be a large, spread out, dominant seventh chord with extensions. There can be as many as 24 audible partials from the lower wound strings. Usually however, we only concern ourselves with the *first 8 partials* which make up what appears to be when written out on a music manuscript grand staff, a large dominant seventh chord which covers three octaves. When you play any particular note of the piano, you of course do not hear what sounds like a chord (one usually only perceives the fundamental) but try this trick to actually hear them: press slowly enough one of the lower notes of the piano, take C2 for example, so that it does not sound and hold it open (using the sostenuto pedal if the piano has one) and play in *staccato* fashion the partial series up to the eighth partial, you will clearly hear each note of the *chord* come eerily out of the open C2 string. Hold open C2 and then strike C3, G3, C4, E4, G4, A-sharp 4 and C5. C2 is the *first partial* or *fundamental* and C5 is the *eighth partial* in this example.

When you play any octave, you will naturally have sets of partials from each string which sound side by side or *coincide*, thus you have what are known as *coincident partials.* But to complicate matters greatly and give the tuner the ultimate and nearly unsolvable problem and challenge, all piano strings bear a phenomenon known as *inharmonicity.* Basically, this means that the upper partials are not exact mathematical multiples of the fundamental tone, as they are thought of theoretically. They are always somewhat sharper than a theoretical partial would be and the higher the partial, the more exaggerated the sharpness. Inharmonicity occurs because a piano's string is made of steel which has a certain stiffness factor to it which distorts what otherwise would fit a theoretical mathematical model.

If there were no inharmonicity as there is none in an instrument such as a pipe organ, all coincident partials would match each other when an octave is tuned so that it has no beat but with piano strings, it is only possible for one set of coincident partials to match at any given time when tuning. When one set of coincident partials match, all others must be and are off or out of tune with each other, to some degree or another, depending on the inharmonicity and how high the partials are. Thus, we have the *types* of octaves; the most commonly used are the 2:1, 4:2 and 6:3 types, although there are other possibilities.

In the 2:1 type octave, the second partial of the lower note is exactly in tune with the fundamental or first partial of the upper note. In the 4:2, the fourth partial of the lower note is exactly in tune with or matches the second partial of the upper note and likewise, with the 6:3 type, the sixth partial of the lower note matches the third partial of the upper. It is also possible that none of the partials match exactly, all may be a slight mismatch but the octave can still have a pleasing or desirable sound, depending on the goal of the tuner.

To begin the fine tuning process, you must first tune your A4 to the pitch source. You start by simply tuning a beatless unison to the fork or other source such as a tone from a metronome that has an A-440 pitch or any other A-440 tone that you consider reliable. Then, to verify that the match is exact, you go down to the note, F2. Now, this is a *fine* tuning procedure, not what you would do for a pitch raise or lowering, something only to verify the ultimate precision and perfection such as when attempting the Tuning Exam or for a public performance or professional recording where the specification on the contract is A-440. Your piano needs to already be nearly perfectly tuned for any rapidly beating interval check or test to be of any useful or practical value.

Play the interval, F2-A4, a M17. You should hear a rapid beat but probably not very rapid and the F2-A4 seventeenth must be wide of being a pure or beatless interval, not narrow. Flatten just *temporarily* the F2 so that it creates a very rapid beat, as fast as you can discern without it becoming an indiscernible blur. Alternately play the F2-A4 M17 on the piano, then the F2-A4 interval from F2 on the piano and the A4 from the pitch source. When both intervals beat *exactly the same*, it confirms that the piano's A4 and the pitch source are exactly the same. It is possible to have an aural accuracy which would measure to within one cent electronically using this test. Accuracy within one cent would score a perfect 100 on the Tuning Exam and would satisfy any contract obligation.

If you hear a slight discrepancy, all you have to do is adjust your A4 up or down by the slightest amount to make the F2-A4 test match exactly. If your piano's F2-A4 is slightly faster than the piano/pitch source F2-A4, your piano's A4 is slightly sharp and conversely, if the piano's F2-A4 is slightly slower than the piano/pitch source F2-A4, the piano's A4 is slightly flat.

Tuning the various *Types* of Octaves

Now, tune the A3 to the A4 and to start, make it as pure or beatless sounding octave as possible. Try sharpening the A3 until you hear a slight beat, indicating the octave is slightly narrow, then flatten it back slightly until it sounds perfectly pure. This will probably be a 2:1 octave. To test for the 2:1 octave, go back down to the F2 you previously used to test the A4 pitch. Again, it must be near where it will eventually be in fine tuning, a wide interval with a rapid beat. But flatten it *temporarily* so that it is very rapid, as rapid as can be without being an indiscernible blur. Play the F2-A3 M10 and then play the F2-A4 M17. If both intervals beat *exactly the same*, the A3-A4 octave is a perfect 2:1 octave. If there is a discrepancy, it is not a perfect 2:1 octave but a wide or narrow one from that point of view.

Now, from a perfect 2:1 A3-A4 octave, flatten again but very slightly the A3 but still retain what sounds like a pure octave with no perceptible beat. This will be a *very* small amount. Now play the F3 and A3 together and listen for a rapid beat. The F3-A3 third must be wide of perfect, not narrow and if your piano is already nearly perfectly tuned, it will beat about 7 beats per second. Now, *temporarily* flatten the F3 as you did previously with the note F2 to make the F3-A3 3rd beat very rapidly, as fast as can be discernible without it becoming a useless blur. Now, alternately play the F3-A3 M3 and then the F3-A4 M10. When both the M3 and the M10 beat *exactly the same,* the A3-A4 octave is a perfect 4:2 type. If the M10 is slightly faster than the M3, the octave is a little wider than a perfect 4:2 which will in fact, be the ultimate goal but more on that later. To learn these differences, try to get these *Equal Beating* tests absolutely perfect so that you will know and understand the difference, one from the other.

Now, from a perfect 4:2 octave, flatten slightly again the A3 until there is a very slow beat between the A3 and A4, just a little more than you might feel comfortable hearing. Now play the A3-C4 m3. If your piano is already nearly perfectly tuned, the A3-C4 m3 will beat very rapidly at nearly the limit of what may be discernable. A m3 is normally about 16 cents narrow in an optimally tuned Equal Temperament and at that spot on the keyboard; the beat is close to being an indiscernible blur. Therefore, you may *temporarily* sharpen the C4 slightly, to make the A3-C4 interval easier to hear. Now, alternately play the A3-C4 m3 and the C4-A4 M6. When both the A3-C4 m3 and the C4-A4 M6 beat *exactly the same,* the A3-A4 octave is a perfect 6:3 octave.

Now, to achieve the optimum compromise between a 4:2 and 6:3 octave, sharpen the A3 very slightly so that *neither* check for a 6:3 or a 4:2 octave tests perfectly. In other words, the test for a 4:2 octave should reveal a slightly faster F3-A4 M10 than F3-A3 M3 and the test for a 6:3 octave should reveal a slightly slower C4-A4 M6 than A3-C4 m3. When you have found the spot for A3 which reveals this slight discrepancy between the 4:2 and 6:3 tests, you will hear that the A3-A4 octave has a very slow beat to it, about one beat in every two seconds. This is now considered the optimum width for the initial A3-A4 octave in Equal Temperament and what is used by most CTE's to set up the Master Tuning for the Tuning Exam.

It is also the equivalent to within a very small and negligible degree of a 4:2 octave plus one cent. This is the width of octave that Dr. Al Sanderson used to obtain the amount of stretch needed for optimal piano tuning when he created the calculation for his Electronic Tuning Device, the Sanderson Accu-Tuner. In this kind of compromise, note that *none* of the coincident partials are in tune or match exactly with each other. This may also be an example of the *whole* octave sound of which Virgil Smith often speaks. This, almost but not quite perfect tuning, is an example of the kind of compromise which is necessary to defeat the problem and challenge of inharmonicity, the way to achieve the finest tuning possible from the modern piano, throughout its entire range.

Also a note of importance, 6:3 type octaves are considered optimum for Bass tuning. The m3 with lower note and M6 with upper note test is useful down to about the note F1. When the octave has a reasonably good sound with just a slight and very slow beat when a single octave is played and both m3 and M6 interval checks beat *exactly the same*, the octaves in the Bass are optimally tuned.

To tune the seventh octave in the high treble in perfect 2:1 octaves with the sixth octave below it, as is required for the Tuning Exam, use the test for the 2:1 octave as described, a M10 below the bottom note and a M17 below the top note. In most other situations however, the seventh octave tuned so that it matches other partials from further down the keyboard is considered ideal, depending upon the perception and taste of the pianist whom we serve. A future article is planned which will discuss and explore all of these options.

In summation, the tests for the 2:1, 4:2 and 6:3 octaves are as follows:

2:1 use the test note a M10 below the bottom note and a M17 below the top note.

4:2 use the M3 below the bottom note and the M10 below the top note.

6:3 use the m3 above the bottom note and a M6 below the top note.

The Equal Beating techniques are attributed to Owen Jorgensen RPT.

The understanding of coincident partials is from Jim Coleman, Sr. RPT and his *Coleman Beat Locater* charts from Fred Tremper RPT.